$$= \left(-\frac{8}{3}+6-4\right)\text{Log}2 - \left[-\frac{1}{9}x^3+\frac{3}{4}x^2-2x\right]_1^2$$
$$= -\frac{2}{3}\text{Log}2 + \frac{8}{9} - 3 + 4 + \left(-\frac{1}{9}+\frac{3}{4}-2\right) = -\frac{2}{3}\text{Log}2 + \frac{19}{36}.$$

B]1/ $u_n = \int_1^2 f_n(x)dx = \int_1^2 (x-1)^n \text{Log}x\, dx$

$$\begin{pmatrix} u'(x) = (x-1)^n & \rightsquigarrow & u(x) = \frac{1}{n+1}(x-1)^{n+1} \\ v(x) = \text{Log}x & \rightsquigarrow & v'(x) = \frac{1}{x} \end{pmatrix}$$

$$u_n = \left[\frac{1}{n+1}(x-1)^{n+1}\text{Log}x\right]_1^2 - \int_1^2 \frac{1}{n+1}\frac{(x-1)^{n+1}}{x}dx$$
$$\Leftrightarrow u_n = \frac{1}{n+1}\text{Log}2 - \frac{1}{n+1}\int_1^2 \frac{(x-1)^{n+1}}{x}dx$$
$$\Leftrightarrow (n+1)u_n = \text{Log}2 - \int_1^2 \frac{(x-1)^{n+1}}{x}dx.$$

a- $(n+1)u_n = \text{Log}2 - \int_1^2 \frac{(x-1)^{n+1}}{x}dx.$

$$\Leftrightarrow \text{Log}2 - (n+1)u_n = \int_1^2 \frac{(x-1)^{n+1}}{x}dx.$$

• $\forall x \in [1;2]$ on a $\frac{1}{2} \le \frac{1}{x} \le 1$

$$\Leftrightarrow \forall x \in [1;2] \text{ on a } \frac{(x-1)^{n+1}}{2} \le \frac{(x-1)^{n+1}}{x} \le (x-1)^{n+1}$$

D'où $\int_1^2 \frac{(x-1)^{n+1}}{2}dx \le \int_1^2 \frac{(x-1)^{n+1}}{x}dx \le \int_1^2 (x-1)^{n+1}dx$

$$\Leftrightarrow \left[\frac{(x-1)^{n+2}}{2(n+2)}\right]_1^2 \le \text{Log}2 - (n+1)u_n \le \left[\frac{(x-1)^{n+2}}{(n+2)}\right]_1^2$$
$$\Leftrightarrow \frac{1}{2(n+2)} \le \text{Log}2 - (n+1)u_n \le \frac{1}{n+2}$$

b-

$$\begin{cases} \frac{1}{2(n+2)} \le \text{Log}2-(n+1)u_n \le \frac{1}{n+2}; \forall n \in \text{IN}^* \\ \lim\limits_{n\to+\infty} \frac{1}{2(n+2)} = 0 \text{ et } \lim\limits_{n\to+\infty} \frac{1}{n+2} = 0 \end{cases}$$

$$\Rightarrow \lim_{n\to+\infty} [\text{Log}2 - (n+1)u_n] = 0 \Leftrightarrow \lim_{n\to+\infty} (n+1)u_n = \text{Log}2.$$

2/a- $S_n(x) = 1 - (x-1) + \ldots + (-1)^n(x-1)^n$

$$= \sum_{k=0}^{n}(-1)^k(x-1)^k = \sum_{k=0}^{n}[(-1)(x-1)]^k$$

Comme $x > 0$ alors $x - 1 > -1$

$$\Leftrightarrow (-1)(x-1) < 1$$